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Verifying Hooke's Law through Longitudinal Standing Waves on a Mechanical Spring

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Abstract: **This paper is a laboratory report on an experiment conducted to verify the Hooke's Law and study the properties of longitudinal waves which are formed on a mechanical spring. This has been done by plotting the elongation of the spring against the applied force. The relation between length of the spring and wavelength of the longitudinal wave has also been verified. Wave propagation velocity was also studied.**

Keywords: **Hooke's Law, mechanical spring, longitudinal waves, standing waves.**

I. INTRODUCTION

This experiment was conducted at the Barnard College Physics Lab, Columbia University under the supervision of Professor Timothy Halpin-Healy. First the spring constant for two springs (heavy and light) was calculated and then the speed of wave propagation on that spring was calculated. This observed speed was verified with the calculated propagation velocity from Hooke's Law.

II. PRE-REQUISITES

When a spring is stretched by a distance x, the restoring force F generated is directly proportional to this applied force:

$F \propto x$

$F = Kx$

Where, K is the characteristic property of the spring called spring constant. A force vs elongation graph will thus be a straight line passing through the origin, the slope of which will give the value of the spring constant.

Wavelength is double of the distance between two successive nodes (places of 0 displacement).

Frequency is the number of vibrations completed in one second.

For the nth harmonic of oscillation, the wavelength λ can be determined as:

$\lambda = 4L/n$

Where, L is the length of the oscillating spring.

For any medium of wave propagation, the propagation velocity C is related to wavelength λ and frequency f as:

$C = \lambda f$

Using Hooke's Law,

$C = (KL/\mu)^{1/2}$

Where, K is the spring constant, L is the length of the spring and μ is the mass per unit length of the spring. [1]

III. EXPERIMENTAL SET UP

Two springs - one heavy spring and one light spring were taken. First, the speed of propagation was experimentally calculated. For this, a mechanical driver on which the spring could be attached using banana plugs was used. The frequency of the generator could be varied and was visible on the screen. The harmonic could be determined by counting the number of nodes, which determined the wavelength using the above-mentioned formula.

Next, the spring constant was experimentally determined. This involved suspending the spring on the wall against a ruler and then suspending different weights on it. The elongation for each case was noted and the slope of the force (the weight of the suspended mass) vs elongation graph gave the spring constant. [2]

The graphs to calculate the spring constant as shown in Fig 1 and Fig 2 were plotted in the lab at Barnard College.

IV. OBSERVATIONS AND CALCULATIONS FOR HEAVY SPRING

A. Experimental Determination of Propagation Velocity

TABLE I: OBSERVATIONS FOR HEAVY SPRING

f_n (Hz)	n	λ (cm)	$C = \lambda f_n$ (m/s)
70.8	15	10.28	7.27
37.4	8	19.275	7.21
41.8	9	17.133	7.16
46.7	10	15.42	7.20
51.5	11	14.01	7.21
57.2	12	12.85	7.35
61.5	13	11.86	7.29
66.3	14	11.01	7.30
75.4	16	9.63	7.26
79.6	17	9.07	7.22
99.7	21	7.34	7.32

C_{avg} = 7.254 m/s

S_c = 0.058 m/s

Propagation Speed of a wave on given slinky (observed) = 7.254 $\pm$ 0.058 m/s

B. Determination of Spring Constant

TABLE II: SPRING CONSTANT FOR HEAVY SPRING

Length (m)	Mass (kg)
0.019	0
0.058	0.02
0.106	0.04
0.129	0.05
0.179	0.07
0.228	0.09
0.253	0.1
0.302	0.12
0.424	0.17

The observations and calculations in Table I and table II were used to plot a graph as shown in Fig. 1

The graph as shown in Fig. 1 was plotted on Kaleidoscope in Barnard College Lab.

$F = KL$

$K = \frac{F}{L}$

Slope = 4.06 N/m

K = 4.06 N/m

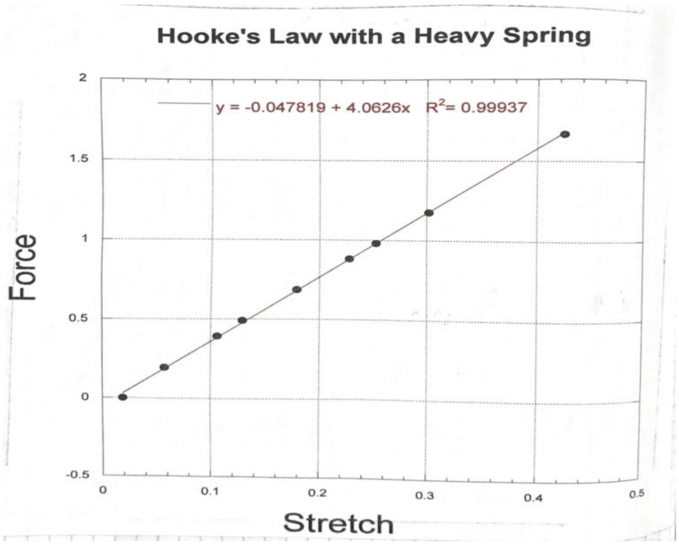

Fig. 1: Graph to Calculate the Spring Constant (Scanned from Lab File)

C. Verification of Formula

m_s = 44.75 g $\pm$ 0.1 g (Using balance)

L_s = 77.1 cm = 0.771 m $\pm$ 0.002 m

$\mu_s = \frac{m_s}{L_s}$ = 0.058 $\pm$ 0.0077 kg/m (Error propagation using calculator)

$C = \sqrt{\frac{kL_s}{\mu_s}}$ (L_s = 0.771 m)

C = 7.35 m/s

D. Error Propagation

$$\frac{\sigma_c}{c} = \sqrt{(\frac{\sigma_L}{L})^2 + (\frac{\sigma_\mu}{\mu})^2}$$

$$\frac{\sigma_c}{7.35} = \sqrt{(\frac{0.002}{0.77})^2 + (\frac{0.0077}{0.038})^2} = 0.09 \text{ m/s}$$

Propagation velocity using Hooke's Law

= 7.35 $\pm$ 0.09 m/s

As 7.35 $\pm$ 0.09 m/s ≈ 7.254 $\pm$ 0.058 m/s

Hooke's law successfully verified for heavy spring.

V. OBSERVATIONS AND CALCULATIONS FOR LIGHT SPRING

A. Experimental Determination of Propagation Velocity

TABLE III: OBSERVATIONS FOR LIGHT SPRING

n	**f (hz)**	**λ (cm)**	**C = λ f (m/s)**
15	56.6	8.97	5.07
17	63.4	7.91	5.01
16	59.5	8.41	5.01
18	66.6	7.47	4.97
19	70.7	7.08	5.01
20	74.9	6.73	5.04
21	78.9	6.4	5.04
22	82.7	6.11	5.05

C_{avg} = 5.025 m/s

S_c = 0.03 m/s

Propagation velocity of spring (Observed) = 5.025$\pm$ 0.03 m/s

B. Determination of Spring Constant

TABLE IV: SPRING CONSTANT FOR LIGHT SPRING

Mass (g)	**Stretch (cm)**
0	0
10	5.3
15	9.9
20	15.2
25	20.8
30	26.3
40	37.3

The observations and calculations in Table III and Table IV were used to plot a graph as shown in Fig. 2

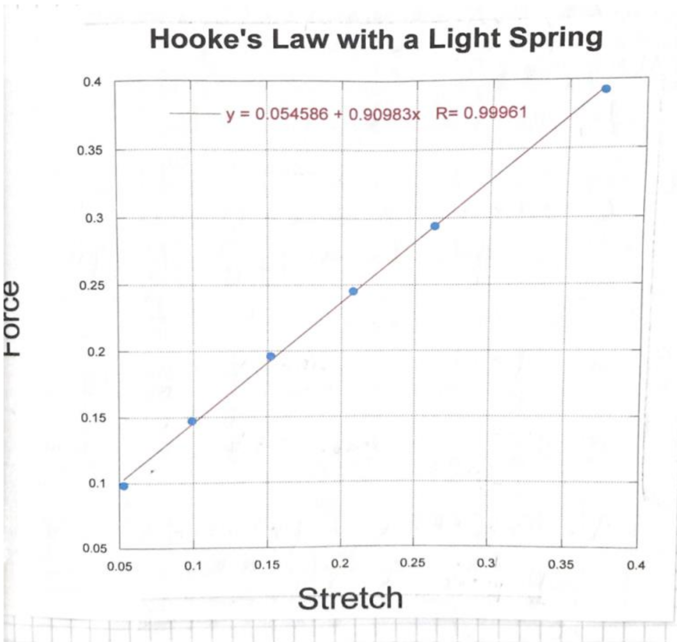

Fig. 2: Graph to Calculate the Spring Constant of Light Spring (Scanned from Lab File)

K= slope = 0.909 N/m

C. Verification of Formula

Mass of spring m_s = 15.85 g = 0.01585 kg $\pm$ 0.0001 kg

Length of spring L_s = 0.673 m $\pm$ 0.002 m

$\mu_{s=}\frac{m_s}{L_s}$

= 0.0235 kg/m $\pm$ 0.0009 kg/m

$$C = \sqrt{\frac{KL_s}{\mu_s}}$$

= 5.10 m/s

D. Error Propagation

$$\frac{\sigma_c}{c} = \sqrt{(\frac{\sigma_L}{L})^2 + (\frac{\sigma_\mu}{\mu})^2}$$

$$\frac{\sigma_c}{5.1} = \sqrt{(\frac{0.002}{0.673})^2 + (\frac{0.0009}{0.235})^2} = 0.02 \text{ m/s}$$

Propagation velocity using Hooke's Law

= 5.1 $\pm$ 0.2 m/s

As, 5.1 $\pm$ 0.2 m/s ≈ 5.025 $\pm$ 0.03 m/s, Hooke's Law successfully verified for lighter spring.

VI. CONCLUSION

Hooke's Law was successfully verified for a light as well as a heavy spring. The error between the calculated and observed values for lighter spring was less.

The spring constants were:

1. 4.06 N/m for heavier spring
2. .909 N/m for lighter spring

The calculated values of C were:

1. 7.35 $\pm$ 0.09 m/s for heavier spring
2. 5.1 $\pm$ 0.2 m/s for lighter spring

The observed values of C were:

1. 7.254 $\pm$ .058 m/s for heavier spring
2. 5.025 $\pm$ 0.03 m/s for lighter spring

The errors exist because the practically calculated wavelength will not be completely accurate. Also, of the springs are stretched 50% they will extend elastic region and the value of K will change and same results will not be obtained.

REFERENCES

[1] Prof. H. C. Verma, IIT Kanpur, "Concepts of Physics", Vol. 2, 2014.

[2] Prof. Timothy Halpin-Healy, "Columbia University Summer Program (2017) in Theoretical and Experimental Physics", Lab Handout, n.d.